THE JOURNAL OF Fixed INCOME

PORTFOLIO MANAGEMENT RESEARCH
By With Intelligence

WINTER 2022

VOLUME 31

NUMBER 3

Occam's Razor for Bond Trade Costs

Vlad Rashkovich and Andrei Iogansen

Bloomberg

JFI.PM-RESEARCH.COM

Bloomberg

Bloomberg, the global business and financial information and news leader, gives influential decision makers a critical edge by connecting them to a dynamic network of information, people and ideas. The company's strength—delivering data, news and analytics through innovative technology, quickly and accurately—is at the core of the Bloomberg Terminal. Bloomberg's enterprise solutions build on the company's core strength: leveraging technology to allow customers to access, integrate, distribute and manage data and information across organizations more efficiently and effectively.

Vlad Rashkovich

Global Head of Quantitative Trading Research at Bloomberg LP

Vlad Rashkovich leads multi-asset Quantitative Trading Research, TCA and Trade Analytics for Bloomberg in New York, where he focuses on introducing novel frameworks and decision support for institutional traders and portfolio managers. In this role, he built a new team of experienced quants to create models and tools for trading stocks, ETFs, baskets, bonds, FX, and futures.

Prior to joining Bloomberg, Vlad was a senior product manager at AllianceBernstein responsible for multi-asset portfolio optimizations and quantitative models' implementation. He also spent time with UBS as an Associate Director and was a portfolio manager for high-net worth clients.

Vlad has been an editorial board member of *The Journal of Trading* and founder of the *Trading BulletIn*, a periodical for 450 traders and portfolio managers on Bloomberg. He frequently speaks at industry conferences and lecturers at Hebrew U, Courant Institute of Mathematical Sciences at NYU and John Hopkins. Vlad has been interviewed by the Institutional Investor for their "Voices of Influence" series and invited to share his views on the future of trading with Chief Investment Officers at U.S. Institute. His research has been published in the *The Journal of Trading*, *The Journal of Fixed Income*, *Best Execution*, *Automated Trader*, and *Bloomberg Markets*.

Vlad earned his bachelor's degree in economics and accounting from the Hebrew University of Jerusalem and his global MBA from Columbia Business School.

Andrei Iogansen

Senior Quantitative Researcher at Bloomberg LP

Andrei Iogansen is a Senior Trading Researcher at Bloomberg where he focuses on quantitative modeling for traders across asset classes. Andrei's expertise sits in formulating and solving mathematical models in quantitative finance, stochastic processes, multi-dimensional optimization, non-linear regression, and algorithmic trading.

Andrei began his career working at the St. Petersburg Nuclear Physics Institute and Harvard University in Quantum Field and String theories. He switched to financial industry as a quantitative analyst by joining Martingale Technologies and subsequently worked with Blue Crest North America, Caxton Associates, SAC Capital Advisors and Millennium Partners.

Andrei earned his M.S. in Theoretical and Mathematical Physics from St. Petersburg State University and a Ph.D in Theoretical and Mathematical Physics from St. Petersburg Nuclear Physics Institute. He is an author of about 70 research papers published in professional journals.

Occam's Razor for Bond Trade Costs

Vlad Rashkovich and Andrei Iogansen

Vlad Rashkovich
is a global head of quantitive trading research at Bloomberg LP in New York, NY.
vrashkovich1@bloomberg.net

Andrei Iogansen
is a senior quantitive researcher at Bloomberg LP in New York, NY.
aiogansen1@bloomberg.net

KEY FINDINGS

- Trade cost in corporate and government bonds can be explained with R^2 = 30%–35% by applying a few of the most powerful factors from bond market microstructure.
- The aforementioned factors exhibit behaviors dissimilar from other asset classes, thus breaking traditional TCA canons.
- Trade cost in bonds has a skewed and unusual distribution (half normal, half Laplace), naturally reflecting dealers' P&L.

ABSTRACT

Inability to accurately project transaction cost is one of the main drags on alpha and performance for bond investors. We introduce a framework for bond trade cost analysis that reflects bond characteristics as well as order information. This framework leverages historical and real-time data to deliver solid explanatory power. The authors goal is to help buy-side traders and dealers to build liquidity trees, while assisting portfolio managers to make investment decisions that include trade costs. We lean on 20 years of experience modeling transaction cost for equities, as well as intimate knowledge of the bond market microstructure. Our work covers investment grade and high yield corporate bonds, issued in USD, EUR, and GBP, as well as government bonds in developed and emerging markets globally.

Trade cost has a very significant impact on investment alpha across asset classes. For equities, Coppejans and Madhavan (2007) show that outperformance of equity funds is halved when transaction costs are considered. According to Perry (2018), 95% of US equity funds underperformed their benchmarks.

For bonds, Dobrescu, Li, and Möttölä (2018) analyzed 25 Morningstar categories, including funds from the US, Europe, Asia, and Africa. They found that net-of-fees median returns were negative in all 25 categories studied for the period 2002–2017.

When the top 20% of these bond funds are selected in each category, the median of the best funds outperforms the underlying benchmark in just 13 out of 25 categories. The median fund outperformed by more than 0.5% in only two categories over a rolling three-year period. None of the categories showed more than 1% outperformance. It is worth noting that compound 0.5% outperformance over three years translates to less than 0.15% annual outperformance. Thus, only the best 10% of bond funds (median of top 20%) and only in 2 out of 25 categories outperform by more than 0.15% a year.

According to the aforementioned research by Dobrescu, Li, and Möttölä (2018), besides fees, transaction cost is the heaviest factor weighing down on performance of bond funds. In contrast to equity indexes, which are often rebalanced annually or quarterly, the majority of bond indexes need to be rebalanced monthly to capture new bonds that are issued and to exclude bonds reaching maturity or being "called," or those no longer complying with rating or term criteria of an index. In the 2014–16 period, for instance, the average turnover rate for the Bloomberg Barclays US Aggregate Bond Index was 40% annualized.

Since we established that transaction costs could account for half of the alpha for the best bond funds and cause an underperformance for an average fund, we decided to study them in depth.

After all, equity investors have been enjoying trade cost estimates in their portfolio optimizers for almost two decades. It is time for bond investors and traders to have transaction cost models as well.

DATA SETS

Data Set 1: Post-Trade TCA Platform

We have used a unique pool of historical transactions from Bloomberg's post-trade TCA platform, with more than 5 million parent orders for corporate and sovereign bonds. More than 250 buy-side firms generated these orders during the period of Jan 1, 2016—Oct 31, 2020.

We have applied filters to remove small odd lots and outliers, keeping the following:

- Placement Size > 1,000 (We have left small sizes to study asymptotic behavior for the size factor, which is not trivial for bonds. See our initial findings below.)
- For corporate bonds, include only bonds issued in USD, EUR, and GBP currencies.
- For sovereign bonds, exclude US government bonds since they are super liquid and measuring trade cost for them would require a more microscopic scale.
- For sovereign and investment grade corporate bonds, include orders with trade cost (see definition of trade cost below) between –200 basic points (bps) and +200 bps in the currency (not spread) space. Upper bound for Bid/Ask spread is also defined as 200 bps.
- For high yield corporate bonds, we keep trades with trade cost between –500 and +500 bps. Upper bound for Bid/Ask spread is also defined as 500 bps.
- Bond prices between 50 and 150.
- Placement type = Market.
- No voice trades were used.

After applying filters, we end up with 2.2 million parent orders: 1.15M for corporates and 1.05M for sovereigns.

Orders can be executed over multiple placements, some taking several days. Multi-day orders add more noise to data since they are affected by price momentum. Therefore, we decided to deal with placements executed the same day. We combine placements for a given bond, firm, side within a trading day, thus generating daily orders. An arrival price for this daily order corresponds to the earliest placement, and execution price is a size-weighted average execution price of all placements. This approach allows us to keep many large orders with their underlying placements intact, while reducing the modeling noise from multi-day transactions.

As an arrival price of a placement we use Composite Bloomberg Bond Trader (CBBT) mid-price (m), since it is a bond-specific real-time benchmark based on executable quotes from all qualified dealers on Bloomberg. To calculate Bid/Ask spreads, we use CBBT bid and ask prices.

We define the trade cost as slippage in basis points 'bps' in the currency (not spread) space.

$$\text{Trade Cost }(\text{Buy}) = 10^4\left(\frac{x}{m} - 1\right) \tag{1}$$

for investor buys, and

$$\text{Trade Cost }(\text{Sell}) = 10^4\left(1 - \frac{x}{m}\right) \tag{2}$$

for investor sells, where x stands for average execution price.

Data Set 2: Bloomberg Venue Data

We have used RFQ quotes and trades from Bloomberg trading venues across various geographies.

In this data set we don't aggregate child orders by client but rather focus on calibration of smaller trades. The smaller trades have the largest noise, so using one venue with consistent time stamps significantly improves the signal.

After applying the same filters as in the aforementioned post-trade TCA platform, we have 3.11 million events left: 1.95 million for corporate bonds and 1.16 million for sovereign bonds. These data cover the span of 14 months from June 2020 to July 2021.

INITIAL ANALYSIS OF BOND TRADE COSTS

Given that equity markets are well studied and have models with a good fit, we decided to start there and to find where bond markets are different. Rashkovich and Verma (2012) showed that the main factors responsible for trade cost in equities are

- Bid/Ask spread
- Participation rate
- Execution time interval
- Size/ADV (average daily volume)
- Stock volatility

NB: We exclude trend from our discussion since for the pre-trade model it is unknown.

It is easy to see that among the top equity factors responsible for transaction cost, bonds lock three out of five factors—participation rate, execution time interval, and ADV. Thus, bonds will naturally require a different approach.

A variety of ideas on how to measure trade cost in fixed income has been summarized well by Sharif et al. (2018). In addition, "imputed round-trip cost" (IRC) introduced by Feldhutter (2012) gained traction. In this approach, researchers try to find buys and sells in the same bond with the same volume within a short period of time. It is a good idea for approximation when the order data are not available.

INITIAL FINDINGS

Below are initial findings from our research that will guide our trade cost modeling for bonds:

a) Asymmetry in trade cost of buys compared to sells
Our research confirms that buys have more impact than sells, similar to findings of Hendershott and Madhavan (2015), Mizrach (2015), and Ruzza (2016). As a reminder, we define trade side from the investor perspective.

b) Large orders are not necessarily more expensive
Odd lots might cost more, especially for buys.
For example, Edwards, Harris, and Piwowar (2007) show that large trades are less costly than small trades.

c) 1% of outstanding is a good proxy for ADV
Since ADV to normalize sizes is not available for bonds, our research shows that using 1% of outstanding is a good proxy, except for newly issued bonds and those close to maturity. Similarly, FINRA's research shows 1% of outstanding is a good proxy for ADV (Mizrach 2015).

d) Grouping for Ratings
While we clearly see correlation between lower rating and higher trade cost, we found delineation between IG and HY to be too crude for trade cost modeling. At the same time, there might not be enough AAA-rated corporate bonds to calibrate the model for them separately.

In our research we have found that all A-rated bonds could be grouped together. BBBs are still IG but should be calibrated separately since they are on the cusp with HY bonds. The rest of Bs could be grouped together, followed by Cs & Ds as a separate group. Non-rated bonds (NR) are a mix of junk and some good companies that have chosen not to pay for rating (e.g., smaller companies with a limited number of bond issuances).

As a result, we used five categories for bond ratings:

- AAA-A
- BBB
- BB-B
- CCC-D
- NR

BUILDING A MODEL

To measure trade cost we need to define an arrival price as our starting point. As we have mentioned above, we have decided to use CBBT mid-price, because it is a bond-specific real-time benchmark based on executable quotes from all qualified dealers on Bloomberg. Since we want our model to cover many geographies, an additional advantage of using CBBT is that it has global coverage.

We have numerous factors to consider for trade cost modeling for bonds, including:

- Side
- Size (non-monotonic)
- Bid/Ask spread
- Bond's outstanding amount

- Rating
- Currency
- Term
- Age
- Time to maturity
- Industry/country

Chacko et al. (2005) found that credit quality, the maturity and age of a bond, the size of a bond issue, industry segment, and provisions such as a call, put, or convertible options all have a strong impact on liquidity.

As William of Occam famously stated: "*Non sunt multiplicanda entia sine necessitate*" (Entities are not to be multiplied without necessity.)

To echo William of Occam, Guo, Lehalle, and Xu (2019) found that the Bid/Ask spread is explicable by other factors including volatility, trading activity, total volume, and years-to-maturity.

As we simplify the model, the main continuous factors are Bid/Ask spread, size, and outstanding, while categorical factors are side, rating, and currency. NB: corporate and government bonds were studied separately. For all other aforementioned factors, adding them to the model resulted in no statistically significant improvement. Essentially, we found that many potential factors mentioned above are highly correlated with Bid/Ask spread, and hence their effect is already captured in the model by the Bid/Ask spread factor.

If we define the cost as "bps" in the currency (not spread) space, our proposed equation for expected trade cost is

$$\overline{Trade\ Cost} = Spread\left[A\left(1-\varphi_1\left(\frac{q}{q_0}\right)\right)+B\varphi_2\left(\frac{q}{q_0}\right)\left(\frac{Size}{V}\right)^{\alpha}+C\right] \tag{3}$$

Here *spread* stands for CBBT Bid/Ask spread, q is order size, $q_0 > 0$ is a scale parameter characterizing sensitivity of trade cost for odd lots, and V is 1% of amount outstanding. As we have mentioned in the initial finding, empirically we have found that 1% of amount outstanding is a reasonable scale for average daily trading volume.

Functions φ_1 and φ_2 monotonically interpolate from 0 to 1 when q/q_0 changes from 0 to infinity. They allow for controlling behavior at the round-lot size interval. A detailed discussion and further research might be warranted on an appropriate choice of φ_1 and φ_2 since they may vary by currency, rating, and asset class while evolving over time.

The first term in Equation 3 in the brackets is responsible for higher observed costs for retail trades. That phenomenon was previously observed in several publications, including Edwards, Harris, and Piwowar (2007) and Harris (2015). It can be interpreted as smaller investors not getting the best prices for their trades and smaller orders from larger firms being routed to retail desks. From the dealer perspective, there is a cost of doing business per transaction, so when this cost is divided by the smaller size, the odd lot might result in the higher relative cost.

It is also important to note that the upswing cost effect for smaller orders is stronger for investor buys. The reason is that the dealer has to buy a round lot and then sell only its fraction as an odd lot to the client. By increasing the cost of the odd lot, the dealers try to recoup the cost of the larger position they had to pay for. When an investor sells an odd lot, the dealers can just place this position on the balance sheet, in case they can't sell it right away. It is worth noting that with increasing electronic trading in bonds, the effect of higher cost for odd lots might erode over time.

EXHIBIT 1

Actual and Estimated Trade Cost vs. Order Size (MM) for Corporate Bonds, Buys, Rating BB-B

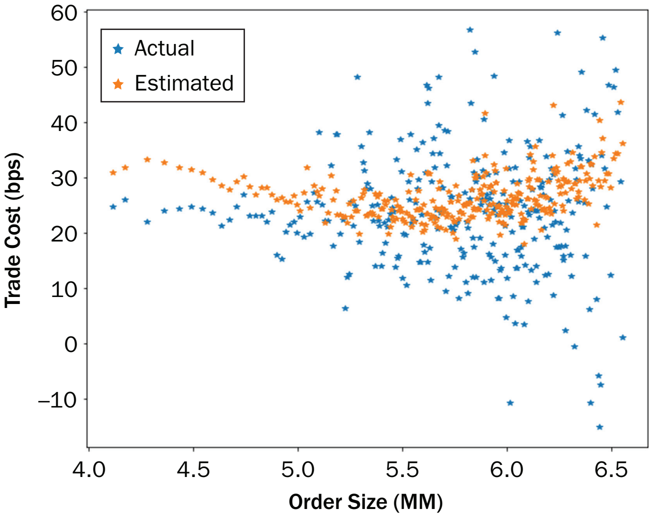
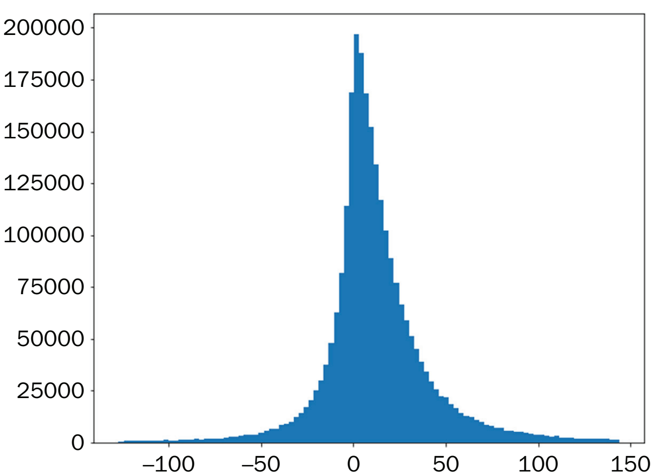


EXHIBIT 2

Trade Cost Histogram for Corporate Bonds



The second term in Equation 3 describes the difficulty of expecting orders as they grow in size. This is a classical TCA factor of trade cost increasing with transaction size. We found that the parameter $\alpha < 1$, and hence, a dependence of trade cost on large sizes is sublinear. Square root of size is an industry proxy for the size factor. Our research for bonds shows even more sublinear relationship corresponding to the power of 0.4.

The third term in Equation 3 is a constant and can be interpreted as a cost of doing business for dealers. It roughly corresponds to the smallest round lot, or so-called social size, where the cost is the lowest. In our analysis, we allow the coefficient *C* to be sector dependent for corporate bonds and country-dependent for sovereign bonds. For sector classification, we use BICS1, which is Bloomberg Industry Classification Standard.

We found that an introduction of sector dependence of C does not add any improvement in correlations between estimated and actual costs.

This is an indirect evidence that sector dependence is already incorporated into Bid/Ask spread. In contrast, we found relatively significant improvement in correlations for normalized trade costs, that is, for ratio Trade Cost/Spread, that supports the above statement. This finding is consistent with conclusions of Hendershott and Madhavan (2015).

For Sovereigns, we found that the effect of country dependence on coefficient C adds on average 3.2% correlation improvement, while ranging between 0% and 10% depending on the currency, rating, and side; it is systematic and statistically significant. These improvements are found both for trade costs and normalized traded costs. This observation implies that the Bid/Ask spread does not incorporate full information on countries that issued those bonds.

For illustration, Exhibit 1 shows actual and estimated Buy Trade Cost vs. order size for corporate bonds with rating BB-B (dots correspond to medians of 300 order bins) for venue data. One can see that dispersion tends to grow with order size. That is an important observation in modeling trade cost, as we will see below.

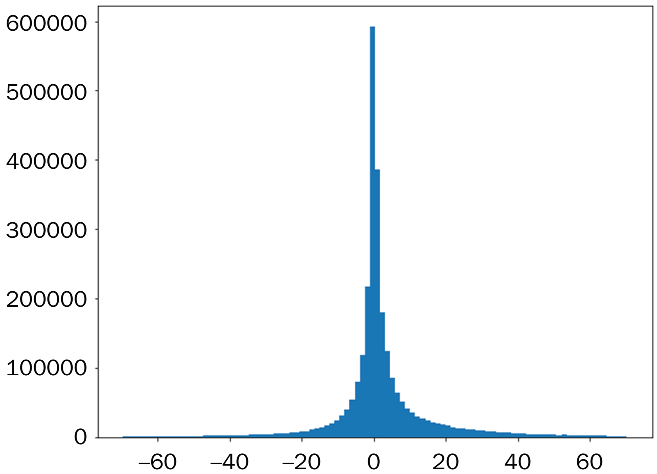
TRADE COST DISTRIBUTION

So far, we have discussed our findings regarding expected trade cost. However, it might be more informative to analyze and model a cost distribution as a function of selected factors. Below we describe our findings for posterior cost distributions conditional on ability to find the required trade size.

Histograms of trade cost within various bond categories traditionally have been approximated as asymmetric Laplace distributions (e.g., Kotz, Kozubowski, Podgórski 2001). For illustration, marginal histograms for corporate and sovereign bonds are shown in Exhibits 2 and 3.

EXHIBIT 3

Trade Cost Histogram for Sovereign Bonds



A detailed analysis shows that a more relevant class of distributions are those with normal left tail and an exponential right tail. As a result, we model trade cost distributions as a combination of half-normal (left tail) and half-Laplace (right tail) that coalesce, or originate, at the mode. More formally we denote trade cost as x,

$$f(x) = \frac{2h}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} \text{ at } x < \mu, \tag{4}$$

$$f(x) = \frac{1-h}{\rho} \exp\left\{-\frac{x-\mu}{\rho}\right\} \text{ at } x > \mu, \tag{5}$$

where

$$h = \frac{\sigma}{\sigma + \rho\sqrt{2/\pi}}$$

Left/right widths σ and ρ are parametrized as functions of the same factors used in the equation for expected trade cost, that is,

$$\sigma = Spread\left[a_1\left(1-\varphi_1\left(\frac{q}{q_0}\right)\right) + b_1\varphi_2\left(\frac{q}{q_0}\right)\left(\frac{Size}{V}\right)^{\alpha} + c_1\right] \tag{6}$$

$$\rho = Spread\left[a_2\left(1-\varphi_1\left(\frac{q}{q_0}\right)\right) + b_2\varphi_2\left(\frac{q}{q_0}\right)\left(\frac{Size}{V}\right)^{\alpha} + c_2\right] \tag{7}$$

The motivation for this ansatz follows from empirical analysis of trade cost distributions in various domains of the factor space: widths of trade cost distributions increase with Bid/Ask spread, and uncertainties in cost increase both for larger order sizes and for smaller order sizes. Typically, minimal uncertainties correspond to social sizes. For illustration, see Exhibit 6.

One can see that in this model, both skewness and kurtosis are nonlinear functions of order size as well as of discrete factors such as rating, currency, and industry, and they do not depend on Bid/Ask spread.

In this ansatz, mode μ is also proportional to Bid/Ask spread since

$$\overline{trade\ cost} = \mu + (1-h)\rho - h\ \sigma\sqrt{2/\pi} \tag{8}$$

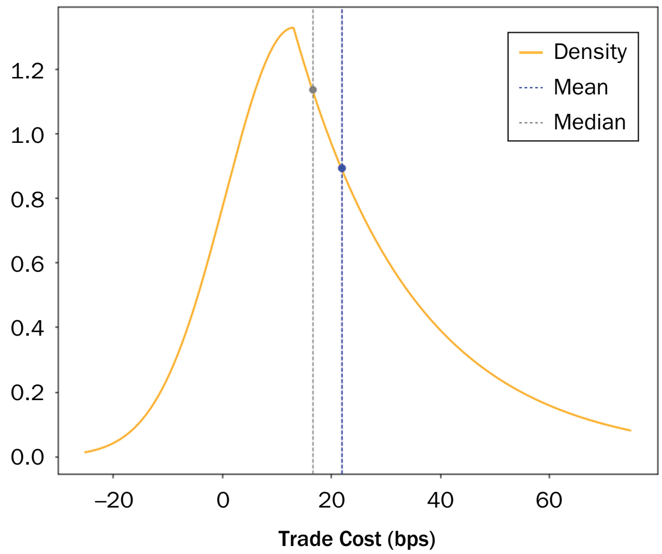
Empirically, we find that $\rho > \sigma$. Therefore, expected trade cost is higher than mode, and median trade cost is between mode and expected trade cost.

This result allows us to describe a trade cost distribution for any given factor values. With such a distribution, one can estimate a probability to trade with cost smaller than any fixed value, given that required liquidity can be found in the market. An example of trade cost distribution density is shown in Exhibit 4.

These findings also imply that mode and median are less sensitive to trade size than mean trade cost as visualized in Exhibit 5. One can also see that the widths of the distribution grows with trade size and spread, and hence with expected trade cost.

EXHIBIT 4

Trade Cost Distribution Density for Corporate Bonds, Buys, Rating AAA-A



NOTE: Currency EUR, Bid/Ask spread 50 bps (in the price space), amount outstanding 700 million, time-to-maturity 5 years, age 2 years, order size 5 MM.

The business reason for such an unusual distribution is that the dealer behavior naturally diverges in trades where they make a substantial profit comparing to a situation where they trade at a loss. When investors need to trade a bond urgently and are ready to pay up for it, they will find a dealer to execute this trade at a premium price, thus creating a right tail that is heavy. Left tail represents a dealer either trying to offload a certain position or aggressively pursuing a bond when there is a taker on the other side. In these cases, the dealer will reluctantly reduce the trade cost because it directly eats into the profit. Thus, the left tail is quickly falling.

In Exhibit 6, we illustrate the behavior of left and right dispersions as functions of order size. One can see that widths of trade cost distributions diverge for large orders and for odd lots as well.

GOODNESS OF FIT

Observed trade costs are very noisy, and the left tail of distribution has negative values, which correspond to trading through the mid, with average costs being positive (see Exhibits 2 and 3). Moreover, the level of noise varies substantially across the factor space. In reality, these histograms correspond to mixtures of distributions corresponding to different market segments. While the trade cost histograms look quite discouraging, it is possible to detect statistically significant signals by analyzing data separately in different market segments and different domains of the factor space. A detailed analysis uncovers a nontrivial dependence of trade cost distributions on factors as discussed above.

Another difficulty is that the data set is very non-uniform with respect to order sizes. Many orders correspond to sizes below round lots. Thus, any global fitting method would suppress contributions of low-populated factor domains unless one artificially overweighs those contributions. Moreover, as discussed above, the trade cost distributions are skewed and heteroscedastic.

To overcome these difficulties, we iteratively use the Migrad algorithm (James and Roos 1975), which earned its reputation in high-energy physics. This approach allowed us to separately fit different domains asymptotically in factor space and combine them without any artificial distortion of data. More precisely, this algorithm was applied to maximize the likelihood of observed events using our ansatz.

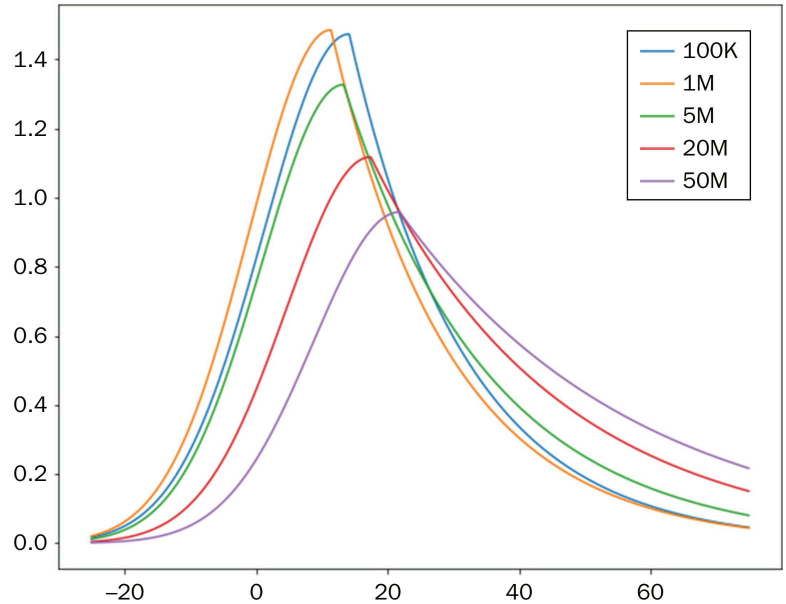
For estimating goodness of fit, one standard method is to calculate correlations between actual and expected values. We should note that correlation coefficients are not always reliable indicators because they converge slowly with sample size and their significance depends on underlying distributions that are typically unknown.

Besides the usual drawbacks, correlation and rank correlation of expected and actual trade costs are not quite adequate indicators because both noise and observed density of data significantly vary across the factor space. Moreover, distributions of coefficients A, B, C have heavy tails, and the error intervals for predicted trade costs depend on order characteristics. In fact, it is more instructive to analyze trade cost distributions conditional of factor values as discussed in the previous section.

While imperfect in use for nonlinear models, R^2 or correlation metrics are common. Therefore, to get an initial feeling of the quality of the model and to compare

EXHIBIT 5

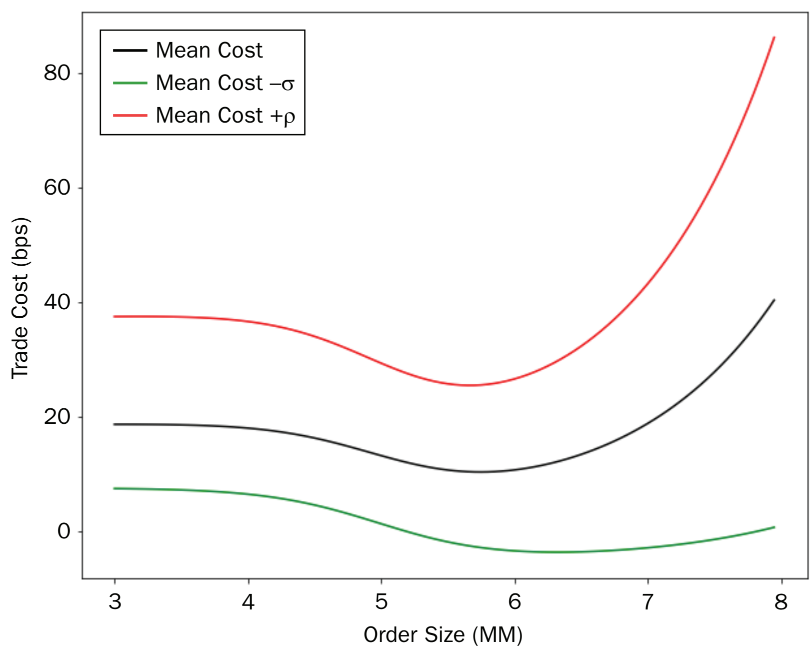
Order Size Dependence of Trade Cost Distribution Density for Corporate Bonds, Buys, Rating AAA-A



NOTE: Currency EUR, Bid/Ask spread 50 bps (in the price space), amount outstanding 700 million, time-to-maturity 5 years, age 2 years.

EXHIBIT 6

Average Trade Cost and Left/Right Deviations vs. Order Size for Corporate Bonds, Buys, Rating AAA-A



NOTE: USD, Bid/Ask spread 50 bps (in the price space), amount outstanding 1 billion, time-to-maturity 5 years, age 2 years.

with findings reported in the literature, we focus here on traditional correlation and rank correlation analysis of actual and modeled trade costs.

It turns out that correlation in various bond subclasses and geographies ranges from 21% to 63%. See Exhibits 7 and 8.

EXHIBIT 7

Correlations of Actual and Modeled Trade Costs for Corporate Bond Market

Currency	Side	Rating	Correlation (%)	Rank Correlation (%)
USD	Buy	IG	45	46
USD	Buy	HY	49	40
USD	Sell	IG	43	29
USD	Sell	HY	34	21
EUR	Buy	IG	52	44
EUR	Buy	HY	63	47
EUR	Sell	IG	44	38
EUR	Sell	HY	32	28
GBP	Buy	IG	55	54
GBP	Buy	HY	61	64
GBP	Sell	IG	23	24
GBP	Sell	HY	24	28

EXHIBIT 8

Correlations of Actual and Modeled Trade Costs for Sovereign Bond Market

Side	Rating	Correlation (%)	Rank Correlation (%)
Buy	IG	59	53
Buy	HY	50	41
Sell	IG	43	28
Sell	HY	42	36

We also calculate rank correlations that are more robust and less sensitive to outliers (though non-parametric). They range in the same intervals, thus confirming the statistical significance of our findings.

It is worth noting that we investigated the ability of sophisticated machine learning algorithms, such as neural networks and Bayesian Monte Carlo methods, to extract a stronger signal with the inclusion of other potential factors. However, the results turn out to be quite unstable with respect to filtering/outliers and out-of-sample selection, and eventually those algorithms do not provide any reliable improvements.

It is interesting to compare our results with those reported in the literature. In particular, Hendershott and Madhavan (2015) show R^2 of 0.05 to 0.10 in modeling trade costs for US investment grade corporate bonds. More specifically, they reported R^2 of 0.10 in their Models 2 and 3 (with a smaller R^2 in their Model 1). This research was focused on odd lot and round lot trades of Investment Grade corporate bonds in the US market. Therefore, the effect of increased costs for oversized trades was not visible.

In contrast, our models cover a much wider range of bonds and include both investment grade and high yield corporate bonds issued in USD, EUR, and GBP. In addition, our model extends beyond corporate bonds and covers government bonds globally across developed and emerging mkets.

Regarding a benchmark, Hendershott and Madhavan seem to use interdealer prices in TRACE. This benchmark might be helpful for post-trade analysis but is unavailable in real-time in most markets to most participants, especially on the buy side. Since our goal is to provide clients with a real-time pre-trade tool, we use Bloomberg's CBBT which, offers a commonly accepted real-time benchmark with global coverage.

When we apply our model for the same market domain of Investment Grade US corporate bonds, correlations are shown in Exhibit 7. One can see that the R^2 values in our model are around 0.18–0.20 (correlation of 43%–45%). When we look at odd lots and round lots only, we get even higher explanatory power of up to $R^2 = 0.30$ (correlation of 55%) in Exhibit 9. When we apply the model to sovereign bonds, we get R^2 of up to 35% (correlation of 59%) as shown in Exhibit 8. Zooming into odd lots and rounds lots for sovereigns reveals similar high explanatory power per Exhibit 10.

LIQUIDITY THROUGHOUT THE TERM STRUCTURE

Liquidity is an important aspect in estimating trade cost and probability of execution. The data show clear dependence of observed daily trade volume on age of bond, for both corporate and sovereign markets.

In Exhibit 11, we show an example of such dependence. The multiplier is defined as daily trade volume normalized by average daily volume for age and time-to-maturity greater than one year calculated separately for each bond.

The estimated multiplier is a result of a nonlinear regression against the ratio age/term. Though trades in the Bloomberg database constitute only a subset of trades in the market, we can reasonably expect it to be representative, and hence, can use it as a good proxy for volume/age dependence in the entire market.

EXHIBIT 9

Correlations of Actual and Modeled Trade Costs for Various Order Sizes for Corporate Bond Market

Currency	Side	Rating	<0.1M	0.1M–0.5M	0.5M–1M	1M–3M	>3M
USD	Buy	IG	57	44	52	55	41
USD	Buy	HY	51	51	52	57	53
USD	Sell	IG	51	40	53	40	22
USD	Sell	HY	41	32	29	30	21
EUR	Buy	IG	57	54	52	48	43
EUR	Buy	HY	52	63	62	66	51
EUR	Sell	IG	49	44	46	45	39
EUR	Sell	HY	37	30	35	37	28
GBP	Buy	IG	64	55	51	55	51
GBP	Buy	HY	49	62	67	65	55
GBP	Sell	IG	41	23	25	25	11
GBP	Sell	HY	14	25	17	43	23

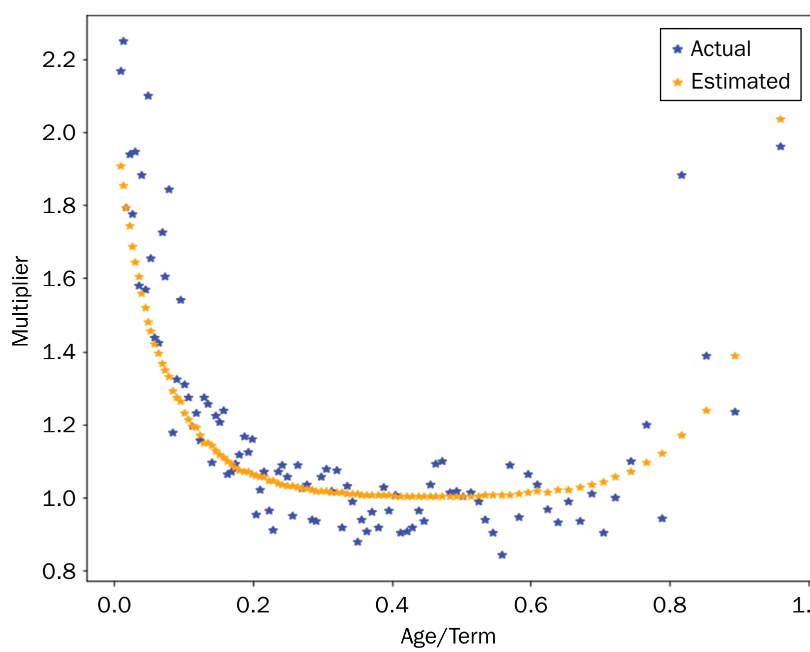
EXHIBIT 10

Correlations of Actual and Modeled Trade Costs for Various Order Size Intervals for Sovereign Bond Market

Side	Rating	<0.1M	0.1M–0.5M	0.5M–1M	1M–3M	>3M
Buy	IG	62	61	58	61	58
Buy	HY	65	53	46	52	48
Sell	IG	53	37	44	46	42
Sell	HY	66	38	44	47	46

EXHIBIT 11

Liquidity Profile for Sells, Rating BBB, Currency EUR



NOTE: Each point represents an average value in a bin of 100 orders.

OUR FINDINGS

Below are the main findings of our research on trade cost for corporate and sovereign bonds. Many of these findings are intuitive for bond traders and our goal is to support traders' experience and intuition with a model that reflects, market dynamics.

Finding #1: The main explanatory factor for trade cost is Bid/Ask spread. After dividing the data by discretionary factors, such as rating, currency, and side, the most influential factor responsible for slippage versus CBBT's mid is Bid/Ask spread. It should make sense to traders that for bonds with similar rating and currency, the bond with a larger Bid/Ask spread will have a higher trade cost.

Finding #2: Buys are more expensive than sells, especially for larger sizes. Side asymmetry of the bond market is the result of bond market microstructure. Dealers taking positions on their balance sheet accrue a cost. If dealers can't offload a trade in the intra-dealer market, then they have to place the acquired bond on their balance sheet. The cost could be even higher if the dealer shorted the bond to sell it to the client. Thus, investor buys are on average more expensive than investor sells, because of a high cost and risk for dealers.

Finding #3: Order size and trade cost relationship is non-monotonic. Odd lots cost more in some markets and for some asset subclasses, especially for investor buys. Trade cost decreases as order size grows to round lot or so-called social sizes. Then the trade cost starts growing in a sublinear manner, for example, when the order size doubles the trade cost increases less than twice.

NB: As a result of electronification of bond trading, the cost of odd lots might be decreasing over time.

Finding #4: Bond's outstanding amount has negative correlation to trade cost. The larger a bond's issuance, the easier it is to source this bond and to trade larger sizes. With all other factors being equal between any two bonds, the one with a larger outstanding amount will have, on average, lower trade costs. We confirm that 1% of the outstanding amount is a reasonable proxy for average daily trade volume. In addition, we found a clear U-shape dependence of observed daily trade volume on the bond's age.

Finding #5: Trade cost differs across five rating groups. In addition to supporting the common wisdom that trading HY bonds is more expensive than IG bonds, even if Bid/Ask spread and outstanding amount are the same, we refined rating groups into five cost categories.

We have shown that, in terms of trade cost, all As trade similarly and thus could be calibrated together. BBBs trade separately and so do the rest of Bs. Cs and Ds could be grouped together, and NRs contain a mishmash of junks bonds and bonds of solid companies that never applied for rating.

Our suggestion is to use four categories and to estimate an appropriate rating for non-rated companies.

Finding #6: Trade cost varies by currency. For corporate bonds, trade cost behaves differently across currencies, given the same Bid/Ask spread, side, size, and outstanding amount. For sovereigns, trade cost is lower for bonds issued in EUR and GBP, compared with USD and other currencies. As we have noted in the data sets section above, we had excluded US government bonds from our analysis.

Finding #7: Market structure varies by currency. For corporate bonds, we see tighter Bid/Ask spreads for EUR denominate bonds compared with USD and GBP. At the same time, on average, it costs less to trade larger sizes for USD bonds versus EUR ones. It might mean that quotes in the EUR market are more indicative, while USD dealers show firmer quotes.

EXHIBIT 12

Out-of-Sample Test Statistics for Rating/Z-Spread Relation Model

Market Segment	Percentage of Estimates of an Exact Rating Group	Percentage of Estimates +/−1 Rating Group
Corporate	59	99
Sovereign	68	97

Finding #8: Corporate and sovereign bonds should be calibrated separately. While the nature of bond trading is consistent and the cost of trading corporate and sovereign bonds can be estimated with Equation 3 above, our research shows that corporate and sovereign bonds should be calibrated separately.

Finding #9: Trade cost has a non-standard distribution. Traders seem to have more leeway in trading bonds than in trading equities, because they can potentially substitute a less liquid bond with a similar bond that is readily available. This flexibility results in many trades executed at a price better than the average cost expected. At the same time if the trader has to execute a bond that is not liquid, the trade cost could be very punishing.

As a result, one can expect an average cost to be higher than the median. This is a skewed distribution combining normal and exponential tails. See Exhibit 4.

Finding #10: Z-Spread is a decent proxy for rating. Many companies are not rated by major rating agencies. By using logistic regression, we found a statistically significant relation between our 4-group rating and z-spread. We used AAA-A, BBB, BB-B and CCC–D groups.

Out-of-sample test statistics are shown in Exhibit 12. We show the percentage of cases in which z-spread predicted exactly the correct rating group, and a case in which we were exact or off by one rating group, for example, z-spread predicts As, while the bond is rated as BBB.

CONCLUSION

Trade cost could be half of your alpha and might be impossible to avoid. However, trade costs should be considered in portfolio construction and investment decisions as well as measured during trade execution.

The goal of this article was to take a step forward in understanding the nature of trade cost for bonds and modeling their impact. This additional transparency in OTC markets could lead to improved market efficiency and trade cost reduction.

The next logical step would be to try minimizing trade costs. We are actively working on quantitative tools that aim to assist bond traders to optimize trading decisions.

ACKNOWLEDGMENTS

We are grateful to Vladimir Markov for his detailed review of this article and numerous suggestions to improve its structure and transparency. Editorial remarks by Eli Jacobowitz offered more clarity and a better flow to this publication.

We are thankful to Bloomberg for its forward-thinking approach and providing us with the latitude to explore and to innovate. Our gratitude goes to the Trading Insights engineering team for agile implementation and continuous support of this model.

REFERENCES

Chacko, G., S. Mahanti, G. Malik, and M. Subrahmanyam. 2005. “The Determinants of Liquidity in the Corporate Bond Markets: An Application of Latent Liquidity.” NYU Working Paper No. FIN-05-037, https://ssrn.com/abstract=1294173.

Coppejans, M. T., and A. Madhavan. 2007. "The Value of Transaction Cost Forecasts: Another Source of Alpha." *Journal of Investment Management* 1 (5): 1.

Dobrescu, M., J. Li, and M. Möttölä. 2018. "Finding Bond Funds That Can Beat Their Benchmarks After Fees." *Morningstar Magazine*, August/September pp. 31–37.

Edwards, E., L. Harris, and M. Piwowar. 2007. "Corporate Bond Market Transaction Costs and Transparency." *The Journal of Finance* 62: 1421–1451.

Feldhutter, P. 2012. "The Same Bond at Different Prices: Identifying Search Frictions and Selling Pressures." *Review of Financial Studies* 25 (4): 1155–1206.

Guo, X., C. Lehalle, and R. Xu. 2019. "Stylized Facts on Price Formation on Corporate Bonds and Best Execution Analysis." https://ssrn.com/abstract=3357789.

Harris, L. 2015. "Transaction Costs, Trade Throughs, and Riskless Principal Trading in Corporate Bond Markets." https://ssrn.com/abstract=2661801.

Hendershott, T., and A. Madhavan. 2015. "Click or Call? Auction versus Search in the Over-the-Counter Market." *The Journal of Finance* 70: 419–447.

James, F., and M. Roos. 1975. "Minuit: A System for Function Minimization and Analysis of the Parameter Errors and Correlations." *Computer Physics Communications* 10: 343–367.

Kotz, S., T. Kozubowski, and K. Podgórski. 2001. "The Laplace Distribution and Generalizations." ISBN: 978-1-4612-0173-1.

Mizrach, B. 2015. "Analysis of Corporate Bond Liquidity." FINRA Office of the Chief Economist.

Perry, M. 2018. *More Evidence That It's Really Hard to 'Beat the Market' Over Time, 95% of Finance Professionals Can't Do It*. American Enterprise Institute (AEI).

Rashkovich, V., and A. Verma. 2012. "Trade Cost: Handicapping on PAR." *The Journal of Trading* 4: 47–54.

Ruzza, A. 2016. "Agency Issues in Corporate Bond Trading." Accessed November 22, 2021 from https://www.aeaweb.org/conference/2017/preliminary/paper/5YfdN6Dz.

Sharif, M., A. Zheng, R. Nambimadom, and R. Mattu. 2018. "Liquidity in Corporate Credit Markets." PIMCO, Quantitative Research.